Locke and Leibniz on the Utility of Formal Logic

> Some eyes want spectacles to see things clearly and distinctly; but let not those that use them therefore say nobody can see clearly without them.
>
> Locke, *Essay* (4.17.4, 678)[1]

> Euclid's 'formalism' is much more like formalism in literature, which focuses on stylistic niceties …
>
> Mueller 1969, 292

§1. Introduction. Formal logic makes a good living as the standard for justifying and criticizing inference (Quine 1982, 45). Yet not long ago formal logic's detractors outnumbered her devotees; from the Renaissance until Russell and Whitehead's *Principia,* philosophers routinely condemned formal logic as scholastic trifling. The *Port-Royal Logic (ca.* 1662) is typical.

> ... there is reason to doubt whether this part of logic is as useful as is generally believed. Man is more likely to err by drawing inferences from false premisses than by inferring incorrectly from true principles; rarely are we led astray by an argument whose conclusion is incorrectly inferred from the premisses. If any man is unable to detect by the light of reason alone the invalidity of an argument, then he is probably incapable of understanding the rules with which we judge whether an argument is valid - and still less able to apply these rules. (Arnauld 1964, 175)

Leibniz did not share *Port-Royal's* sentiment; he regarded syllogistic as one of the "finest inventions to have been made by the human mind" *(N.E.* 478).[2] But the Cartesian and empiricist traditions found formal logic useless and even pernicious. Descartes rejected formal logic "lest our reason should go on holiday" (Descartes 1972, 32), while Locke argued that it can never achieve more than native rustic reason. Commentators *(e.g.,* Passmore 1953, Hacking 1973 and Gaukroger 1989) have found much to recommend Leibniz, but little of value in the anti-formalists. I argue that Locke's position, properly understood, is impervious to Leibniz's criticisms and superior to Leibniz's account of demonstrative reasoning.

§2. The standard account of anti-formalism. Passmore's 1953 study of Descartes and the British empiricists gives an outline of the anti-formalist critique and concludes by rehearsing its death scene. Passmore argues first that Locke's critique is in the Cartesian and not the Baconian tradition, which holds that all demonstration is inductive. Subsequently, he argues that interest in formal logic was revived once philosophers recognized that its replacement, the Cartesian methodological ideal, was doomed (552-3). That failure became apparent with the failure of Mill's *System of Logic.* Descartes complained that syllogistic was impotent for discovering new truths, and, more importantly, he thought his *Rules* contained such a method.

---

1 Reference is to book, chapter and section of Locke's *Essay.* The page number of the Nidditch edition is also given.

2 Reference is to the page number of the Academy edition of Leibniz's *Nouveaux essais sur l'entendement humain.* The translation is by Remnant and Bennett.

But Passmore cannot make his case against anti-formalism by associating it with the fruitless tradition of the methodological approach to truth.

Locke accepts methods for discovering truths, but as a hope for the future. When method is an issue Locke mentions algebra and expresses his optimism that similar methods can be found *(e.g.,* 4.3.18, 549). Furthermore, he never endorses Descartes' *Rules*. Since the *Essay* holds that methods for discovering truth are themselves mostly undiscovered *(e.g.,* 4.3.6, 540 and 4.17.7, 680), the failure of the Cartesian method shouldn't affect Locke's claim that formal logic achieves nothing more than "native rustic reason." There is, as well, an anomaly to including Locke in a tradition that eventuates in Mill's *Logic,* for Mill bans intuition, the *a priori,* and inconceivability as concealing habits of mind untested by experience *(Logic* 2.5.5-6). Locke, however, presumes the offending notions in his rejection of syllogistic. Evidently, we must evaluate Locke's theory on its own terms. The analysis will lead to a more plausible account of the renewed interest in formal logic, one motivated by developments in nineteenth century mathematics (§9).

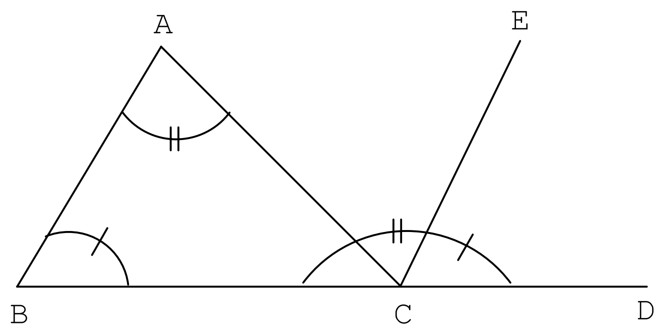
§3. Locke on knowledge and reasoning. Knowledge, says Locke, consists in "the perception of the connection and agreement or disagreement and repugnancy, of any of our ideas" (4.1.2, 525; cf. 4.17.2). For him, intuition—the immediate grasp of the agreement of ideas—is the foundation of knowledge, and one cannot demand greater certainty than that of intuition without showing "that he has a mind to be a sceptick" (4.2.1, 531). Thus, formal logic is a viable enterprise only for one who, "depressing and discrediting nature," fails to give the knowing faculties their due (4.17.4, 678).

Reasoning enlarges knowledge by perceiving the connection of ideas in a proposition through the connection of the intermediate ideas that constitute its demonstration (4.17.2, 668-9). Demonstrative knowledge arises from the intuitive by means of two sub-faculties, sagacity and illation (*ibid*.). Sagacity discovers intermediate ideas, which enable the mind to make connections with other concepts. Illation then arranges the intermediate ideas "as to discover what connexion there is in each link of the chain whereby the extremes are held together."

Locke gives several examples of reasoning (*e.g.*, 4.2.2, 532; 4.17.4, 673), but his paradigm case is a proof of *Elements* I.32.

> Thus, the mind being willing to know the agreement or disagreement in bigness between three angles of a triangle and two right ones, cannot by an immediate view and comparing them do it: because the three angles of a triangle cannot be brought at once, and compared with any other one, or two, angles; and so of this the mind is fain to find out some other angles to which the three angles of a triangle have an equality; and finding those equal to two right ones, comes to know their equality to two right ones. (4.2.2, 532)

According to Locke, I.32 involves a single intermediate idea, say, the augmentation of triangle ABC by extending C to D and constructing CE parallel to AB.



<BCA, <ACE, and <ECD, arranged as they are, show the connection between two right angles and the three angles of ABC, because <ACE = < BAC, and <ECD = <ABC. We must not be misled by the apparent simplicity of Locke's illustration. In the first place, the agreement seems to be a literal resemblance, and this suggests that Locke is ignoring entailment relations. He's not, though, as the example involving self-determination shows (4.17.4, 673). Second, we should not suppose that Locke thinks that the proof depends on perceiving sensibly an exact equality between the angles of ABC and the three angles that compose the straight angle BCD. Surely he understands that a traditional geometric proof begins by *letting* some inexact empirical figure *be* some exact mathematical figure.[3] Here too, he recognizes that demonstration is a conceptual matter, resting upon entailment relations. Finally, the use of a single intermediate idea is problematic, as it may not be obvious that <BAC= <ACE and <ABC= < ECD. These equalities rest upon prior proofs (Euclid I.29) and Locke is sketchy about the role of prior proofs. I would suggest that he supposes that the diagram in I.32 embodies the diagram of I.29, and so the inference goes through.

The proof of I.32 that Locke has in mind recapitulates centuries of geometrical practice, a practice that continues today. It is the practice of informal proof, i.e., proof that does *not* depend upon the forms of the constituent statements. In this instance the proof depends upon general (non-metric) features of an accompanying diagram and rules of geometrical inference, some of which are previously proved. *Formal* geometric proofs are nowadays part of mathematical practice. But they did not appear until the late 19th century, more than 150 years after Leibniz's death. We shall see in §9 that the formalization of geometry, *pace* Leibniz, was not motivated by concerns about certainty and necessity. Moreover, we shall see that informal practice continued to play a crucial role.

According to Passmore, by failing to distinguish truth from validity, anti-formalists can't explain important inferential phenomena (550). He continues,

> On this way of looking at things, the unit of thought is a relation of agreement between two ideas. The distinction between subject and predicate vanishes as of no significance... and so does the distinction between universal and particular propositions ... reasoning is the intuition of resemblances; and "fallacy" simply consists in asserting that there is a resemblance where there is none. (551)

[3] See Sherry 2009 for an analysis of the use of inexact, empirical diagrams in proofs of exact mathematical propositions.

Not all these doctrines follow from failing or refusing to distinguish truth from validity, as Locke's account of reasoning can distinguish universal from particular *(cf.* Pap 1958, 50).[4] Hume may conceive of reasoning as the intuition of resemblances, but Locke is not guilty of this over-simplification.[5]

My intention is not to dispute Passmore's charge, but to evaluate its seriousness. According to the charge, Locke is powerless to deal with an argument like

(*) All freshmen are enrolled in logic

All logic students have an exam tomorrow

So, all freshmen have an exam tomorrow,

whose validity is a separate matter from its soundness. Though everyone is inclined to say that (*)'s conclusion follows whether or not it is true, Locke cannot account for this phenomenon, since there is no intrinsic connection between being a freshman, being enrolled in logic, and having an exam tomorrow. The formalist, however, easily explains the phenomenon in terms of a formal component of understanding, say

If all Fs are G's and all G's are H's, then all F's are Hs.

Unless Locke allows a formal component to knowledge, the force of arguments like (*) remains inexplicable.

Locke does not address arguments like (*), presumably because such cases are irrelevant to a theory of knowledge limited to mathematics and portions of ethics and physics. But his brief chapter "Of Particles" could be construed to include logical particles like "all." He treats particles as stylistic elements of discourse that "signify the connection that the mind gives to ideas or propositions with one another" (3.7.1, 471). Thus "all" indicates that the speaker regards one idea as entailing another. Locke could allow that our inclination to say that (*)'s conclusion follows from its premises is due to an analogy with cases in which conceptual connection obtains.

One could also extrapolate from cases of conceptual or empirical agreement to what would have to be the case if such and such terms were connected. This is well on the way to formal logic, but Locke would question whether such extrapolation would help in the pursuit of knowledge. Indeed, even in the course of testing an hypothesis by means of its observational consequences, it is not necessary to invoke formal schemata. Here too, one reasons informally as in traditional geometry; indeed, the reasoning is generally possible in virtue of a mathematical representation of the situation.[6] Thus cases like (*) do not illustrate an unquestionable benefit to developing and inculcating formal logic.

---

[4] They can be construed as assertions of the compossibility of ideas. Thus, 'some men are philosophers' asserts that the ideas of man and philosopher do not disagree in the sense of excluding one another, and 'some men are not philosophers' asserts that these ideas do not agree in the sense of the former entailing the latter.

[5] The agreement of ideas occurs in several ways, says Locke, revealing their various relations and habitudes (4.1.5, 526; 4.1.9, 529).

[6] For an instructive example, see Toulmin's discussion of shadow casting (1960, 23ff.).

Locke does not deny formal relations; he concedes that...

> syllogistic form...shows that if the intermediate idea agrees with those it is on both sides immediately applied to, then those two remote ones or, as they are called, extremes, do certainly agree. (4.17.4, 674)

But this does not require formal schemata, for he adds

> ...therefore the immediate connexion of each idea to that which it is applied to on each side, on which the force of the reasoning depends, is as well seen before as after the syllogism is made, or else he that makes the syllogism could never see it at all. (ibid.)

The formalist and anti-formalist disagree on how one explains the force of argument. For the formalist, the force of argument depends upon both the capacity to recognize an argument as an instance of a valid schema and the capacity to recognize the truth of its premises. The anti-formalist locates the force in the capacity for recognizing the agreement of ideas, and so it depends on the content of ideas, not their formal presentation. We shall return to this disagreement after explications of Locke's critique of formalism (§§4-5) and Leibniz's defense of it (§§6-10).

§4. Locke against formalism. The dangers of formalism are discussed most fully in 4.7, "Of Maxims," which considers how formal truths "influence and govern our other knowledge" (4.7.1, 591). The primary targets of Locke's attack in 4.7 are the magnified maxims of the scholastics - the principle of identity and the principle of contradiction. But Locke complains that syllogisms as well as maxims are useful only in scholastic dispute, which encourages the mind to mistake verbal trifling for real knowledge.

Locke does not regard the maxims as formally valid schemata, whose substitution instances are, therefore, true. In contrast to formal logicians, he understands the maxims to be epistemological equals of what are usually understood to be their instances. Thus 'a circle is a circle' is not strictly an instance of the formal truth 'whatsoever is, is;' both propositions express the agreement of ideas, only the idea of being is more general than the idea of circle (4.7.4, 594). Locke flattens the logical landscape, for he does not distinguish more general truths on which less lofty ones depend.[7]

Locke presents a dilemma for maxims: If our words have a clear signification, then maxims are not necessary; and if our words lack clear signification, then maxims serve to confirm us in our mistakes. In support of the first horn he claims that an examination of scientific treatises shows that maxims play no role.

> And I would be glad to be shown where any such science erected upon these, or any other general axioms is to be found: and should be obliged to anyone who would lay before me the frame and system of any science so built on these, or any such like

[7] This is not to suggest that he sees no priority relations between propositions. Magnified maxims are made plausible by their particular "instances" (4.12.3, 640).

> maxims that could not be shown to stand as firm without any consideration of them. (4.7.11, 598)

Locke is not merely pointing out the absence of 'whatsoever is, is' from Euclid's *Elements* or Newton's *Principia*; the claim is the stronger one that there is no role for maxims. Thus, Locke asserts that the third axiom of the *Elements*—equals taken from equals, the remainders will be equals – is irrelevant. Once the mind is presented with a pair of equal lengths from which equal lengths are removed, one does not require an axiom to see or justify that the remaining pair are equal. The result of the particular construction is as clear and certain as the maxim upon which it allegedly rests. Thus the appearance of maxims at the beginning of the *Elements* has pedagogical but not epistemological significance.[8] The discoverers of theorems don't need them, and although maxims can act as a familiar touchstone of the face of unfamiliar truths, careful reflection would serve as well (4.7.11, 599-600).

The pernicious side of maxims, the other horn of the dilemma, emerges from a description of the role actually played by formalism.

> But the method of the schools, having allowed and encouraged men to oppose and resist evident truth, till they are baffled, i.e., till they are reduced to contradict themselves, or some established principle; 'tis no wonder that they should not in civil conversation be ashamed of that, which in the schools is counted a virtue and a glory; viz. obstinately to maintain that side of the question they have chosen, whether true or false, to the last extremity; even after conviction. (ibid., 601)

Maxims exist, then, to bring an end to potentially interminable disputes. But the need arises only from the perversity of those unwilling to admit agreement and disagreement which intuition plainly reveals - those with a mind "to be a sceptick." This perversity is possible only with a certain abuse of words.

Book III of the *Essay*, "Of Words," has been extensively criticized. Twentieth century philosophy of language begins by recognizing the naïveté of Locke's "Words ... stand for nothing but the ideas in the mind of him who uses them" (3.2.2). But despite its psychologism, the primary thesis - traditional philosophy thrives on the abuse of language – is difficult to dispute nowadays. In criticizing scholastic philosophy, Locke contrasts the use of native rustic reason, wherein truth is the main objective, with the abuse of language, wherein victory is the main objective. Here the abuse consists in giving "new or unusual significations" to familiar terms or in using "new and ambiguous terms" (*cf*. 3.10.6, 493). While this abuse continues unchecked, success in the disputational art is mistaken for knowledge.

The use of words lacking clear signification is encouraged through appeal to formalism. Even though they are unnecessary, maxims may be useful since, owing to custom, "the very naming of them is enough to "satisfy the understanding" (4.7.11, 603). But their innocuous use depends upon the clarity of the ideas that render them unnecessary. When clear ideas are absent from discussion, especially in disputes of metaphysics, maxims exchange the role of eliminable expedient for the role of confirming us in our mistakes. Even though it lacks clear ideas,

[8] In fact, historical investigation has show that it is mistaken to read the early books of the *Elements* as having an axiomatic structure (Seidenberg 1975).

metaphysics, for example, can present formal coherence as the mark of knowledge and demonstration. The result, says Locke, is a system of merely verbal propositions, and formalism is chiefly responsible for the dignity of such systems.

§5. Locke against syllogistic. Locke patterns his treatment of syllogistic after his treatment of maxims. He stresses the scholastic background and claims syllogistic is chiefly useful for dealing with wranglers, those willing to deny their native lights for the sake of victory in dispute (4.17.4, 675). He claims further that the natural light suffices in situations where syllogistic seems to called for (*ibid*.). These remarks are taken for granted rather than argued. But similarity in strategy, tone and language show that Locke's critiques of maxims and syllogisms are cut from the same anti-formalist cloth.

Locke's main argument against syllogistic appeals to the presuppositions for its use. He maintains that syllogistic, once it has been presented with an inference, is no help in determining whether "the mind has made this inference right or no." His rationale depends on the status of the middle term.

> in neither case is it syllogism that discovered those ideas [i.e., middle terms], or showed the connexion of them; for they must be both found out, and the connexion everywhere perceived, before they can rationally be made use of in syllogism; unless it can be said, that any idea, without considering what connection it has with the two other, whose agreement should be shown by it, will do well enough in a syllogism, and may be taken at a venture for the medius terminus, to prove any conclusion. (4.17.4, 672-3)

Locke's observation is a nice counterpoint to logicians who insist on 'discovering' unstated assumptions that enable one to represent an argument as an instance of a valid inference pattern (e.g., Pospesel 2000, 206-7). Clever individuals realize it's always possible to reformulate an argument as a *modus ponens* in which the 'discovered' premise is a conditional whose antecedent is the premise of the original argument and whose consequent is the conclusion of the original. This strategy is unsatisfying, of course, because the new premise fails to provide any additional insight. When there is a question whether the inference "be made right or no," the 'discovered' premise merely shifts the target of the question from the inference to the truth of a conditional; but the difficulty is exactly the same, 'could such and such be true while so and so is false?' By the same token, the rational application of syllogism depends on recognizing the right middle term, which, in turn, presupposes that one already sees the connections between the extremes and the middle. Locke contends, then, that formalization serves only to obscure difficulties that may lie in the argument.[9] Once attention is drawn to formal correctness, the

---

[9] One might think this criticism bears only upon the use of syllogisms in evaluating enthymemes. Sherry *1991* argues that the application of formal techniques to arguments with apparently no unstated premises still presupposes the ability to see informal logical connections. Briefly, formal techniques are mostly inapplicable without prior use of paraphrase because natural language arguments virtually never exhibit the overlap of terms necessary to see an argument as an instance of a valid pattern. But paraphrase itself rests on the ability to find 'the right middle term', that is the term that enables us to see an argument as an instance of a formally valid pattern. Therefore, it is still problematic whether formal apparatuses suffice to show the connection between premises and conclusion. There is only one sort of argument whose evaluation obviously requires formal techniques, viz., the puzzles created specifically by logicians to show the virtue of their techniques. Venn notes that these puzzles "are seldom forced on us in any practical way" (Venn 1881, xxx). But this is the exception that proves Locke's contention that formal

sense of propositions is put in soft focus, and reason goes on holiday (4.8.5-7, 612ff.; 3.11.24, 521).

§6. Formalism and the natural order. Locke's antipathy toward formalism results, Leibniz says, from a failure to see that

> ...we are not concerned here with the sequence of our discoveries, which differs from one man to another, but with the connection and natural order of truths, which is always the same. (N.E. 412)

Wilson 1967 echoes the same distinction.

> ...this is not to say that "derivation" of one truth from another can have no sense for Locke, but that it can have no sense wholly independent of "the history of our discoveries." (354)

Do these considerations reveal the value of discerning a formal component to knowledge? Euclid's *Elements* is a carefully ordered treatise, in spite of the fact that it avoids proofs based on formal considerations (Heath 1956, I, 256). It is ordered by the ends it seeks to achieve. For instance, the propositions in Books I and II are all aimed at the final proposition of II, to construct a square equal to a given rectilinear figure (II.14). If we work backwards from this proposition, we find increasingly simple constructions and theorems. Yet this sequence is plainly not the only means of achieving the end that orders the propositions of *Elements* I and II. Mathematicians who had tried to eliminate the parallel postulate were aware that it could be derived from various assumptions, including I.32 itself (Heath 1956, I, 204ff.). The possibility of alternative arrangements hardly reveals an undesirable subjectivity in the *Elements*. Rather, the fact that Euclid's ordering remained untouched for so long indicates that his sense of proper order resonated with subsequent generations.

The development of the *Elements* is natural in the sense that it moves from what is easier to grasp to what is more difficult to grasp. Not much is known of the development of the material in Books I and II, primarily because it emerged at the dawn of civilization, ca. 3500 B.C. (Seidenberg 1977). But it is plausible that something like Euclid's order is reflected in the order in which more complex theorems and constructions were discovered. Why, then, is Leibniz concerned to distinguish the sequence of discoveries from the natural sequence of truths? Perhaps the concern arises from the distinction between discovery and justification. Discovery has no logic to it, the story goes, especially when it comes to mathematics. Empirical (or quasi-empirical) evidence, analogies, or even an odd sequence of events can lead to a mathematical discovery, while the validity of that discovery must be independent of such considerations. But this means only that in order to elevate a conjecture to truth, one must demonstrate the conjecture from previously established truths, perhaps with the help of a newly discovered intermediate idea. Aesthetic considerations may lead one to look for a simpler or more natural proof, but *pace* Leibniz and Wilson, the key to a more natural ordering of truths does not require recognizing a formal component to knowledge. This will become clearer below as we contrast

---

techniques are unnecessary in the usual conduct of reasoning. Leibniz himself provides an interesting illustration of this in §9 below.

Leibniz's ideal of order, reducing propositions to identity by substituting *definiens* for *definiendum*, with the manner in which geometry finally appealed to statement form as the medium of demonstrative reasoning.

§7. Leibniz on knowledge and reasoning. The different epistemologies of Locke and Leibniz are illustrated by their respective treatments of geometry. Whereas the geometrical axioms are superfluous for Locke, for Leibniz the same principles are tenuous and still in need of foundation. Instead of finding the axioms trifling, Leibniz urges

> ... the importance of demonstrating all the secondary axioms which we ordinarily use, by bringing them back to axioms which are primary, i.e. immediate and indemonstrable; they are the ones ... I have been calling 'identities.' (N.E. 408; cf. 452)

Apparently, Euclid's axiomatization falls short of Leibniz's ideal of demonstrative knowledge. The failure is aired again when Leibniz chides Euclid for giving a worthless definition of 'straight line', and so having to rely on imagination.

> Euclid, for instance, has included in his axioms what amounts to the statement that two straight lines can meet only once. Imagination, drawing on sense-experience, does not allow us to depict two straight lines meeting more than once, but this is not the right foundation for a science. And if anyone believes that his imagination presents him with connections between distinct ideas, then he is inadequately informed as to the source of truths, and would count as immediate a great many propositions which really are demonstrable from prior ones. (N.E. 451; cf. 75 and 370)

Whereas Locke sees epistemology as the illumination of the actual practice of mathematicians and physicists, Leibniz sees epistemology as the exposition of an ideal, largely unattained. Locke presumes that the working mathematician possesses at least a modicum of well-founded knowledge; for Leibniz that individual, strictly speaking, possesses almost none.

While for Locke, one either sees or fails to see a conceptual connection, Leibniz insists upon gradations.

> So you see, sir, that what you and your friends have said about the connection of ideas as the genuine source of truths needs elucidation. If you are willing to be satisfied with seeing such connections confusedly, you will weaken the rigour of demonstration; Euclid did incomparably better by reducing everything to definitions and a small number of axioms. But if you want this connection of ideas to be exhibited and expressed distinctly, you will have to avail yourselves of definitions and axiomatic identities, as I require. (N.E., 452)

Leibniz sees three grades of rigor, depending on the character and role of axioms. The thoroughgoing informalist has no use for axioms; the mathematician desirous of rigor uses axioms "whose evidence can be seen only confusedly," while the formalist uses only axiomatic identities. What is not clear from this distinction is whether the propositions proved unrigorously are certainties or something less. In some places Leibniz leans toward deductivism, whereby one proves only that if the axioms are true, so are the theorems (e.g., N.E. 450). Elsewhere he observes that mathematicians are capable of drawing necessary conclusions (Leibniz 1956a, 893-6). Here is a tension in Leibniz's formalism: Mathematicians are somehow capable of certainty without actually grasping its source.

The demon that stands in the way of certainty is imagination: "Euclid was ... obliged to rest content with certain axioms whose evidence can be seen only confusedly, by means of images." Leibniz repeatedly accuses Locke of trusting images as the arbiter of mathematical truth (e.g., N.E. 451-2). But Locke distinguishes general ideas from images (e.g., 4.7.9, 596 and 4.3.19, 550-1), and he should do this, because mathematical demonstration has never depended upon accurate diagrams. Hence Leibniz's case rests on an unsympathetic view of the ideas whose connection through intuition the informalist claims to see. This is not entirely Leibniz's fault, as Locke does little to clarify what such ideas would be if not images (cf. N.E. 261).[10] But Leibniz's attempts to convict mathematicians of an unhealthy reliance upon imagination are still not helpful. Mathematicians are guilty as charged only if there can be found a proposition accepted for a time on the strength of imagination and later rejected on the strength of understanding. Leibniz's only example of being led astray by imagination is the case of inferring that lines that continually approach one another must eventually meet (N.E. 452).[11] This 'error' seems never to have been a problem for the mathematical community, and it is surely open to Locke to deny the inference from continual approach to intersection.[12] Without examples of difficulties arising from imagination, Leibniz's charge of confusion is, therefore, insubstantial. When we turn to Leibniz's replacement for intuition, reduction to formal identities, the case for formalism becomes even more tenuous.

§8. Virtual Knowledge. According to Leibniz, formal identities are necessary constituents of knowledge. In discussing maxims he writes

> ...we use these maxims without having them explicitly in mind. It is rather like the way in which one has potentially in mind the suppressed premises in enthymemes. (N.E. 76)

But a suppressed premise is necessary to an enthymeme's cogency, and so Leibniz is claiming that Locke fails to recognize the source of mathematical truth.

> So we shouldn't here be contrasting the axiom with the example, as though they were different truths in this respect, but rather regarding the axiom as embodied in the example and as *making the example true*. (N.E. 413) (my emphasis)

[10] This can be done by explicating the having of an idea as the mastery of a concept (cf. Sherry 2009).

[11] Leibniz characterizes this as an induction, whereas Locke carefully separates agreement of ideas from induction. In this example Leibniz is thinking of curved lines that approach one another asymptotically, not the non-Euclidean scenario in which straight lines approach one another asymptotically.

[12] At least one historian sides with Locke's casual attitude: "In Euclidean geometry, conceived as the description of an intuitively grasped truth, precautions to avoid falsehood are really unnecessary." (Mueller 1981, 5)

On this view, a piece of informal reasoning cannot *eo ipso* constitute a chain of necessary connections; if the conclusion is grasped as certain or necessary, this is because the demonstration embodies and colors a sub-structure of formal logic.[13] But how does one show that the sub-structure must be there to recognize the certainty of a mathematical truth?

Leibniz is attracted to a causal role for formal truths because they explain the necessity and certainty of favored truths. Locke countenances most of the same truths but feels no compunction to explain their necessity or certainty. Locke's stance is not due simply to a lack of curiosity. To carry off the formal analysis that Leibniz envisions, a large stock of definitions is necessary; for his strategy is to show that substitution of *definiens* for *definiendum* will result, in a proposition of the form *A* is *A*. As in much revisionist philosophy, the feasibility of such reductions is at best a promissory note. But even if one were to produce the reductions that the analysis requires, the question of their validity remains. On what basis should one decide upon an adequate analysis of 'straight line,' for example? Surely one criterion will be whether or not the analysis allows one to infer the usual informally known results. In other words, the analysis will rest upon the original way of knowledge, the visible agreement of ideas. Furthermore, as we shall now see, the developments which brought formal logic into mathematical research were not aimed at explaining the necessity or certainty of mathematics.

§9. Formal logic, necessity and certainty. Leibniz regards logical laws as "laws of good sense set in writing" (N.E. 480). Their utility lies in helping "natural good sense:"

> For when natural good sense undertakes to analyze a piece of reasoning without help from the art [of logic], it will sometimes be in a little difficulty about the validity of the inferences—finding for example that the reasoning involves some mood which is indeed sound but which is not in common use. (N.E. 481)

Leibniz subsequently offers no example - let alone an unproblematic one - to illustrate the value of syllogistic (*cf*. Leibniz 1956c, 759). This absence recalls his failure to provide a convincing example of being led astray by imagination. Moreover, when Leibniz himself analyzes Locke's proof of God's existence - a piece of reasoning that is genuinely apt to be confusing—he employs no formal techniques (N.E. 436; *cf*. 4.10.2). Locke's error involves a quantifier shift fallacy, one of the best examples of a fallacy demonstrated by predicate logic. Ironically, Leibniz shows that such a fallacy can be made transparent to common sense without a formal apparatus.[14] He proceeds by reflecting on the significations of the expressions involved and giving counterexamples. Locke may have been overconfident of the power of reason to avoid fallacy, but Leibniz's own practice does little to suggest that formal logic is the remedy for reason's failings.

---

[13] Leibniz proposes that syllogistic can be reduced to a calculus based on identities (Leibniz 1951, *26ff.).* Thus the difference between the principle of contradiction (a version of identity) and syllogistic theory is unimportant.

[14] Similar problems appear in early nineteenth century analysis, with the discovery/invention of pathological functions (*e.g.,* in Cauchy's attempt to prove that the limit of a convergent series of continuous functions is itself continuous). Here too the error was spotted without a formal apparatus *(cf.* Lakatos *1976*, 128 *ff*). The apparatus was still fifty years in the future.

Another rationale for syllogistic is Leibniz's recollection of controversies where "mutual understanding began only after we had resorted to formal arguments to sort out our tangle of reasoning" (N.E. 481; *cf*. Leibniz *1956c*, 763 (note 145)). Though a canonical language is one route to clarity, it is not obviously the best. The prescriptions in Locke's *Essay* surely compete with traditional syllogistic as an avenue toward mutual understanding of discourse (cf. Buickerood 1985). One remedy Locke proposes is fixing the senses of expressions (3.11. 2 *ff*.). Agreeing upon syllogistic form itself presupposes fixing senses, and once senses are fixed, we are left to ask why rustic reason is not sufficient.

Leibniz offered a third avenue of apology, once again without the help of concrete examples.

> I myself am of the opinion that mathematics, history, and other subjects should be learned before an extensive mastery of logic, for how can one order one's thoughts who has never thought of much? But, once provided with a store of good ideas, one can survey and measure them, and, with the help of the order that is uncovered in them, one can all the more readily discover something new. (Leibniz 1956c, 764)

It is not obvious that Leibniz is plumping for syllogistic here. The context of the passage suggests that "logic" may refer to the Ramist technique of division and subdivision, which Leibniz earlier praises for ordering his thoughts (757). But if syllogistic is what Leibniz has in mind, then the formalization of geometry in the late 19th century, by Pasch and others, seems to provide an important illustration. These thinkers did manage to remove intuition from the discipline, and we have been beneficiaries of this accomplishment. But these contributions follow upon a century of geometric innovation that, in crucial respects, runs counter to Leibniz's conception of logical foundations.

The nineteenth century saw the emergence of non-Euclidean and projective geometries. Although non-Euclidean geometry has provoked the bulk of philosophical discourse about geometry, it is really just one aspect of a more general trend, the pursuit of geometry as a symbolic system rather than a body of theorems about extension. Nagel 1979 argues that projective geometry was the source of this trend. Desargues, in the seventeenth century, conducted the first systematic investigations of projective geometry, though contemporaries neglected his work. He introduced points at infinity, where parallels intersect, in order not to have to treat parallel lines differently from non-parallels. Monge and Poncelet, in the nineteenth century, continued this approach by introducing further imaginary points of intersection. Such moves permit proofs of very general theorems that replace groups of theorems about the special cases that fall under a general theorem (Nagel 1979, 201). The gain in generality led Poncelet to conceive of geometrical discourse as an abstract calculus rather than a representation of some subject matter. In support of this approach Gergonne introduced the notion of *implicit* definition, according to which the meanings of geometrical terms were found not in an external subject matter but were given by the ways in which the terms were used in the axioms. This, in turn, prompted Grassman to construe geometry as lacking a subject matter and lacking truth, except for logical truth (216 ff. and 222 ff.). These developments were geometries only by analogy.

Leibniz thinks geometrical truths are reducible to identities by substituting *definiens* for *definiendum*. That strategy presumes a stock of *explicit* definitions, obtained by analysis of individual geometric terms. Thus Leibniz contends that Euclid could dispense with imagination only by analyzing geometric concepts. Far from making possible such analyses, the revolution that freed geometry from intuition gave up the quest for deeper, explicit definition in favor of a plurality of axiom systems in which a geometric concept can appear; for instance, hyperbolic straight lines satisfy Euclid's first four postulates just as much as ordinary, Euclidean straight lines do. This results, in some sense, in an analysis of a geometric concept, but not in Leibniz's sense of a *definiens* substitutable for a *definiens*. Geometry freed itself from intuition not because it might be a Cartesian demon but because it wished to extend a language developed with intuition to cases where there were no intuitions, in other words, to cases outside informal mathematics. A formal calculus is the best we can do under these circumstances. When language is extended beyond the reach of intuition, that is, beyond our natural inclinations to recognize conceptual connections, there formal logic is useful.

Leibniz refers to attempts by Herlin and others to formalize Euclid's geometry (Leibniz *1965*, 8; *cf.* N.E. 361). They present a perfect opportunity to detail advantages that accrued from the labor undertaken; none are forthcoming, and the lack of a relational logic suggests their project was doomed. In sum, the more plausible rationale for formal logic is found in mathematical developments that come after Leibniz and Locke, developments nurtured by a desire to extend mathematics to new territory rather than to explain its necessity and certainty.

§10. Formal Logic and Right Helps. So far Leibniz's defense of formal logic seems not to warrant his praise of it as one of the "finest inventions to have been made by the human mind." At one point he acknowledges, "that the scholastic form of argument is usually inconvenient, inadequate and poorly handled" (N.E. 483). This suggests that he may be applauding some theory other than traditional syllogistic, and in fact this is the case: Leibniz uses "syllogism" to cover "also sequences of syllogisms and everything that [he has] called formal argument" (*ibid*.). "Formal argument" in this wider sense denotes not only algebra and infinitesimal analysis, but also "a well drawn-up statement of accounts" (N.E. 478-9). The attraction of these practices is an art of infallibility, which consists in reducing problems to calculations that promise to exclude errors. Understood in this sense, formal logic is not the target of Locke's polemic, but rather the sum total of the right helps of art, which he often praises (4.3.18, 549; 4.17.7, 680).

This connection with Locke is not lost on Leibniz, who brings Philalethes to remark that right helps of art constitute formal logic in the wider sense. In that light, the controversy perhaps languishes; both agree on the usefulness of right helps of art. But Leibniz cannot plead his case for formal logic by grouping it with algebra and analysis. He needs to show that formal logic – a calculus of statement forms – confers a substantial benefit, by organizing and grounding the rest. Kneale and Kneale 1962 argue that Leibniz sees just such a role for syllogistic:

> [In New Essays Leibniz] describes Aristotle's doctrine as 'one of the most beautiful discoveries of the human spirit,' and says it is an 'art of infallibility' which can be developed into 'a sort of universal mathematics.' (322)

Universal mathematics covers all cases of formal argument; it consists in a universal characteristic and, on the basis of the characteristic, a rational calculus developed using combinatorial techniques. The Kneales make a persuasive case that syllogistic played a major role in Leibniz's conception of a characteristic (*ibid*., 322*ff*.). In particular the doctrine that the predicate is contained in the subject of all true propositions provided a methodological guide as Leibniz attempted to formulate the various ways that ultimates could be combined. For example, it led Leibniz to analyze relational properties like 'grandparent' as conjunctions of attributive facts. By granting that universal mathematics is dictated by the structure of syllogistic, the Kneales explain how Leibniz pays homage to Aristotle while disparaging the scholastic use of syllogistic.

If universal mathematics were viable, Leibniz would have an easy reply to Locke's arbitrary middle term objection. Locke pointed out that the right middle term is necessary for a syllogism to have force, and that we recognize the right term only by recognizing its connection with the extremes. Universal mathematics undermines this argument. For, given a questionable inference, the characteristic could discern the ultimate notions involved and then the calculus would show whether the consequence relation held. We have already seen (§8), however, that the analyses required to obtain the ultimate notions actually rest upon the intuitions that constitute the informal practice. Without assuming the viability of universal mathematics, it's unclear how Leibniz proposes to meet the arbitrary middle term objection.

Outside of Leibniz's commitment to implicit knowledge, it is difficult to see the influence of syllogistic throughout the history of demonstrative science. Locke sees no connection between right helps and the apparatus of syllogistic, and this conception fits better with the subsequent advance of science. Rather than improving informal practice, formalization simply leads to new problems concerning the *correctness* of proposed formalizations. Their solution leads back to the informal theory, as Lakatos's historical investigations have made apparent.

> Does this mean that [… in a …] fully formalized postulate system it is impossible to have any counterexample? Well, it is certain that we won't have any counterexample formalizable in the system, assuming the system is consistent; but we have no guarantee at all that our formal system contains the full empirical or quasi-empirical stuff in which we are interested and with which we dealt in the informal theory. There is no formal criterion as to the correctness of formalization. (Lakatos 1987, 66-7)

§11. Recent defenses of Leibniz. Hacking *1973* and Gaukroger *1989* consider the competing philosophies of proof of Descartes and Leibniz, and both find more merit in the latter: "Leibniz knew what a proof is - Descartes did not" (Hacking *1973*, 175). Their strategies consist in contrasting Leibniz's deep understanding of and fascination with formal proof to Descartes' sense of proof as a regrettable instrument of clear and distinct perception. The implication of their work, particularly Gaukroger's, is that the entire anti-formalist tradition is incoherent. Had these authors paid attention to Locke's views, they might have reached different conclusions.

Hacking's essay explains the sterility of modem philosophy of mathematics in terms of concepts of proof which filled a seventeenth century epistemological need, a need which no longer exists. Hacking's foil is, interestingly, Wittgenstein.

> ...Wittgenstein ends up with a dilemma that is essentially Leibniz-Cartesian. On the one hand he suggests, in quite the most radical way, that mathematical 'truth' is constituted by proof, and on the other he is obsessed by just the intuitions that so impressed Descartes. Hardly anyone thinks he has achieved a synthesis of these notions. (188)

Hacking's claim rests on an untenable dichotomy, Leibniz's formalism and Descartes' anti-proofism (Hacking's term). Locke's informalist program is a third option, and one that erases the alleged tension between proof and intuition. I am not maintaining that Wittgenstein is a neo-Lockean, but it is relevant that both are concerned with mathematical practice rather than its reconstruction. Moreover, it is not surprising to find Wittgenstein accusing mathematical logic of deforming mathematical thought (1978, V, 48). Locke did not feel the scruples that (according to Hacking) led Descartes to exchange proof for a veracious God. Thus there is a respectable place in Locke's philosophy for informal proof. Hacking's account does less than justice to his foil.

Gaukroger's *Cartesian Logic* is a critical study of Descartes' views of inference, but its results are intended to apply to thinkers like Locke. In the midst of defending Leibniz from the Cartesians, Gaukroger turns explicitly to Locke (*1989*, 90-2). Gaukroger sees Descartes and Leibniz as offering different solutions to avoiding errors in proof. Descartes proposes "compacting" inferences into instantaneous intuitions and Leibniz proposes replacing proofs with mechanical procedures (70). Locke, who is content with chains of informal inferences forged from memory and intuition, and aided by diagrams and ciphers, would find both programs motivated by excessive skepticism (*cf*. 4.2.14, 537). Given modest faith in our faculties, Locke meets the challenge raised by Gaukroger on behalf of formal logic.

Gaukroger argues that Descartes was influenced by algebra to reject synthesis and so deduction (*1989*, ch. 3). He contrasts this with Leibniz's famous formal proof that 2+2 = 4, which "goes right to the core of [Descartes'] rejection of the idea that deductive or synthetic demonstrations show nothing" (89-90). What follows is an analysis of the proof, which is supposed to persuade Locke (*sic*!) of the value of formal demonstration. But before we consider the proof we should observe that the thesis that "deduction or synthetic demonstrations show nothing" is ambiguous. We have stressed that deductions can be formal or informal. Descartes, perhaps, had no use for either sort. Locke, however, finds the formal but not the latter to be useless. Once we observe this distinction, Gaukroger cannot defend Leibniz's preference for formal deduction.

Gaukroger acknowledges that the point of Leibniz's demonstration that 2+2 = 4 cannot be to show the less self-evident on the basis of the more self-evident (91). Rather the point of the demonstration concerns truth.

> ... while the self-evidence of the proposition does not depend on the demonstrations, its truth does. Its truth depends on the demonstration in the sense that 2+2 = 4 is true if and only if 2 = 1+1, 3 = 2+1, and 4 = 3+1 are true and if Leibniz's Law (and the associativity of addition) holds. ... Two propositions may be equally self-evident yet the truth of one may presuppose and not be presupposed by the truth of the other. What Locke has assumed is that if something is a self-evident truth it is an independent truth. Leibniz's demonstrations show that this is not the case. (91-2)

This argument does not establish the value of formal over informal deduction. For Locke, a truth is dependent just in case it is not among the intuitively known truths, and he begins from a larger inventory of intuitively known truths than Leibniz. This is a plausible account of dependence, and the fact that Leibniz can present a different one, one in which '2+2 = 4' is dependent, will not impress anyone not already committed to a foundational role for formalism. Again, Gaukroger accepts Leibniz's argument because he presumes that demonstration is formal demonstration, and so ignores different ways in which truths might be ordered.

The fact that Locke's discussion of reasoning fits better with actual practice, and the fact that Leibniz requires *both* the fallibility of intuition *and* the viability of universal mathematics to establish the utility of formal logic, count heavily against formal logic as the foundation of reasoning. When questions of consistency and entailment are directed to discourse from politics, religion, philosophy, and even mathematics, native rustic reason is the *de facto* standard of correctness.

David Sherry
Northern Arizona University

References

Arnauld, A. 1964 *The Art of Thinking,* trans. J. Dickhoff and P. James, Indianapolis, IN (Bobbs-Merrill Co.).

Bolton, M. 1990 "Leibniz and Locke on the Knowledge of Necessary Truths" in *Central Themes in Early Modem Philosophy*, Indianapolis (Hackett).

Buickerood, J. 1985 "Locke and the Rise of Facultative Logic," *History and Philosophy of Logic 6*, 157-90.

Descartes, R. 1972 *Rules for Directing the Mind in Philosophical Works*, vol. I, trans. E. Haldane and G. Ross, Cambridge (Cambridge University Press).

Gaukroger, S. 1989 *Cartesian Logic: An Essay on Descartes' Conception of Inference*, Oxford (Clarendon Press).

Gergonne, J. 1818-19 "Essai sur la Theorie des Definitions," *Annales de Mathematiques Pures et Appliques 9*, 1-35.

Hacking, I. 1973 "Leibniz and Descartes: Proof and Eternal Truths," *Proceedings of the British Academy 59*, 175-188.

Heath, T. 1956 *The Thirteen Books of Euclid's Elements*, New York (Dover).

Kneale, W. and M. 1962 *The Development of Logic*, Oxford (Clarendon Press).

Lakatos, I. 1976 *Proofs and Refutations*, Cambridge (Cambridge University Press).

__________ 1987 "What does a mathematical proof prove?" in *Mathematics, Science and Epistemology, ed. Worrall and Currie*, Cambridge (Cambridge University Press).

Leibniz, G. 1951 "Principles of a Logical Calculus" in *Leibniz Selections*, ed. Weiner, New York (Scribners), 26-8.

__________ 1956a "On What is Independent of Sense and of Matter" in *Philosophical Papers and Letters*, trans. and ed. Loemker, Chicago (University of Chicago Press), 888-898.

__________ 1956b "Letter to Herman Conring" in *Philosophical Papers and Letters*, trans. and ed. Loemker, Chicago (University of Chicago Press), 284-93.

__________ 1956c "Letter to Gabriel Wagner" in *Philosophical Papers and Letters*, trans. and ed. Loemker, Chicago (University of Chicago Press), 753-769.

__________ 1965 "Reflections on knowledge, truth, and ideas," in *Monadology and Other Philosophical Essays* trans. P. Schrecker and A. Schrecker, New York (Macmillan), 3-10.

__________ 1981 *New Essays on Human Understanding*, ed. P. Remnant and J. Bennett, Cambridge (Cambridge University Press).

Locke, J. 1975 *An Essay concerning Human Understanding*, ed. P. Nidditch, Oxford (Clarendon Press).

Mill, J. 1973 *A System of Logic*, in *Collected Works*, vol. 7, Toronto (University of Toronto Press).

Mueller, I. 1969 "Euclid's Elements and the Axiomatic Method," *British Journal for the Philosophy of Science 20*, 289-309.

__________ 1981 *Philosophy of Mathematics and Deductive Structure in Euclid's Elements*, Cambridge, MA (MIT Press).

Nagel, E. 1979 "The Formation of Modern Conceptions of Formal Logic in the Development of Geometry" in *Teleology Revisited*, New York (Columbia University Press).

Pap, A. 1958 *Semantics and Necessary Truth*, New Haven (Yale University Press).

Passmore, J. 1953 "Descartes, the British Empiricists, and Formal Logic," *Philosophical Review 62*, 545-553.

Pospesel, H. 2000 *Propositional Logic*, Upper Saddle River, NJ (Prentice Hall).
Quine, W. 1982 *Methods of Logic*, Cambridge, MA (Harvard University Press).
Seidenberg, A. 1975 "Did Euclid's Elements, Book I, Develop Geometry Axiomatically?" *Archive for History of Exact Science 14*, 263-295.
__________ 1977 "The Origin of Mathematics," *Archive for History of Exact Science 18*, 301-342.
Sherry, D. 1991 "The Inconspicuous Role of Paraphrase," *History and Philosophy of Logic 12*, 151-66.
__________ 2009 "The Role of Diagrams in Mathematical Arguments," *Foundations of Science 14*, 59-74.
Toulmin, S. 1960 *Philosophy of Science: An Introduction*, New York (Harper).
Venn, J. *1881*, *Symbolic Logic*, London (Macmillan).
Wilson, M. *1967* "Leibniz and Locke on First Truths," *Journal of the History of Ideas 28*, 347-366.
Wittgenstein, L. 1978 *Remarks on the Foundations of Mathematics, 2nd ed., rev.*, Cambridge, MA (MIT Press).
Yolton, J. *1970 Locke and the Compass of Human Understanding*, Cambridge (Cambridge University Press).